Asclepio. Revista de Historia de la Medicina y de la Ciencia
65 (2), julio-diciembre 2013, p018
ISSN-L:0210-4466
http://dx.doi.org/10.3989/asclepio.2013.18

ESTUDIOS / *RESEARCH STUDIES*

RENAISSANCE SOURCES OF JUAN PÉREZ DE MOYA'S GEOMETRIES[1]

M. Céu Silva

Centro de Matemática da Universidade do Porto (CMUP)
Seminari d'Història de la Ciència da Universidade Pompeu Fabra
mcsilva@fc.up.pt.

Received: 14 mayo 2012; Accepted: 5 abril 2013.

Citation/Cómo citar este artículo: Silva, M. Céu (2013), "Renaissance Sources of Juan Pérez de Moya's Geometries", *Asclepio* 65 (2): p018. doi: http://dx.doi.org/10.3989/asclepio.2013.18

ABSTRACT: Juan Pérez de Moya became known in the Iberian Peninsula because of the publication of his *Arithmetica Practica, y Speculativa*; however, he was also the author of an interesting geometric work which has not yet been disseminated. In this article we focus on the characteristics, evolution and methodology of the texts which Moya devoted to geometry, revealing the goals he intended to reach with each of them, the subjects he dealt with, and the works he used as references. The comparison we established with some of the best known mathematical works of the Renaissance allowed us to identify the main sources of Moya, and to evaluate the extent of their impact on the preparation of Moya's texts. The revisions implemented by Moya in his geometries, until reaching an effective compromise between practice and theory, reaffirms him as a practical geometer with theoretical concerns, and as the main disseminator of Euclid's *Elements* in XVI[th] century Spain.

KEY WORDS: Juan Pérez de Moya; Sixteenth century; Practical Geometry; Tartaglia; Mathematical works of the Renaissance.

FUENTES RENACENTISTAS DE LAS GEOMETRÍAS DE JUAN PÉREZ DE MOYA

RESUMEN: Juan Pérez de Moya, conocido en la Península Ibérica por la publicación de la *Arithmetica Practica, y speculativa*, fue también el autor de un interesante trabajo geométrico, aún no suficientemente conocido. En este artículo nos centramos en las características, evolución y metodología de los textos de geometría de Moya para dar a conocer los objetivos que quería alcanzar con cada uno de ellos, los temas que desarrolla y los trabajos que le han servido como referencia. La comparación que hemos establecido con algunas de las mejores obras matemáticas del Renacimiento nos ha permitido identificar las principales fuentes de Moya y evaluar el impacto que tuvieron en la preparación de sus textos. Las revisiones que Moya introdujo en su obra para llegar a un compromiso efectivo entre los aspectos teórico y práctico lo reafirman como un geómetra práctico preocupado con la teoría y como el principal divulgador de los *Elementos* de Euclides en la España del siglo XVI.

PALABRAS CLAVE: Juan Pérez de Moya; Siglo XVI; Geometría Práctica; Tartaglia; Obras matemáticas del Renacimiento.

1. INTRODUCTION

Juan Pérez de Moya was born in Santisteban del Puerto, in 1513, and died in Granada in 1596. He was not a mathematician of first magnitude, because his works do not contain original contributions; nevertheless, Valladares Reguero (Valladares Reguero, 1997, p. 378) notes that Moya's texts, both the mathematical and the religious ones, show didactical and popularization character, and López Piñero (López Piñero, 1979, p. 176) stresses their importance with regard to mathematical knowledge[2].

Moya began his mathematical production in 1554 with the *Libro de Cuenta*. In this work, he introduces the reader to the knowledge of elementary arithmetic concepts, such as operations with integers and fractional numbers, the rules for reckoning without feather (*reglas para contar sin pluma*) and he explains how to make the conversion between different currencies. The work opens with a dialogue, which serves as motivation to the learning of arithmetics. The *Libro Segundo de Arithmetica* published in Salamanca in 1557 is smaller than the *Libro de Cuenta*. It treats the so-called "minor art", which includes the rule of three and the concept of proportion, the "Castilian cuenta" (which still uses the Roman numerals), and teaches the currency exchange. The *Compendio dela regla dela cosa* was printed in the following year. This is the first text that Moya dedicates to the study of algebra, or the "high art", and it is also the first Iberian printed work with exclusively algebraic content[3]. The importance that Moya attaches to algebra is clear in the following words: "the beautiful content of which cannot be more highly recommended" (*cuja hermosa materia no se puede mas altamente encomendar*).

The three works just mentioned were then reordered originating the *Arithmetica practica, y speculatiua*, published in Salamanca in 1562 (Baranda, 1998, p. X). The diversity of topics that the book covers made it attractive for audiences with different skills, needs, and interests such as the simple arithmetical procedures or the study of equations. This work includes a little chapter on geometry, whose main objective is to introduce some procedures for land surveying.

Having collected in the *Arithmetica practica, y speculativa* all the theoretical and practical arithmetical knowledge indispensable in a diffusion mathematical work, Moya then turns to the study of geometry. He publishes in the years of 1567 and 1568 the *Obra intitulada Fragmentos Mathematicos* in two books (Pérez de Moya, 1567) and (Pérez de Moya, 1568), dedicating the first volume to the geometrical material[4]. His intention was to treat in detail all the subjects related to the Liberal Arts. But the high cost of the printing did not allow him to materialize in full his intention. Therefore he was forced to select some topics of practical geometry, which he considers to be important tools to those who study other sciences[5].

Five years later, Moya revisited all the mathematical material that he had already produced, and after improving (correcting some existing lapses) and expanding it, he publishes it in a voluminous work, the *Tratado de Mathematicas* (Pérez de Moya, 1573). This work consists of three volumes (the first book is on Arithmetic, the second is on Geometry and the last is on Astronomy, Cosmography and Natural Philosophy). It is a kind of "suma" of all the mathematical knowledge of that time.

Based on the themes developed in the *Tratado de Mathematicas*, Moya composed two more works adapting their content to specific audiences. One is entitled *Arithmetica de Moya intitulada manual de contadores* (Pérez de Moya, 1582). The other is Moya's last mathematical work, the *Principios de Geometria de que se podran aprovechar los estudiosos de Artes Liberales, y todo hombre que su officio le necessitare a tomar la regla y compas en la mano. Con el orden de medir, y dividir tierra* (Pérez de Moya, 1584)[6]. The titles of these two works clearly show its recipients.

2. CONTEXTUALIZATION

The first printed geometrical treatise in the Spanish literature is Juan de Ortega's *Compusicion de la arte dela arismetica y Juntamente de Geometria* (Lyon, 1512); it is devoted especially to Arithmetic, the geometrical questions coming forth almost exclusively as applications of the arithmetic rules which have been introduced. This work of Ortega had several reprints and a great spread in Spain; however even the improved version that was published in the second half of the sixteenth century, the *Tractado subtilissimo de arismetica y geometría* (Granada, 1563), still presents a very rudimentary Geometry made up of numerical examples of everyday life, mostly of which concern land surveying[7]. We do not know any Spanish work of the first half of the sixteenth century entirely devoted to Geometry. According to Consolación Baranda (Baranda, 1998, p. XIV) the Spanish texts printed in this period are confined to mercantile computations, and only in the reign of Felipe II was there an increasing interest on the various branches of Mathematics.

In sixteenth century Europe there are some unavoidable names in the study of Geometry and its applications, such as Finé, Bovelles, Ringelberg, Peletier, Apiano, Dürer, and others, whose works were directed to specific areas of that science. One of the most important works, both for the diversity of geometrical themes presented and for the relations established between practical Geometry and its theoretical foundations, is Tartaglia's *General Trattato* (Venezia, 1556-1560). In the titles of the three geometrical books of

Asclepio, 65 (2), julio-diciembre 2013, p018. ISSN-L: 0210-4466. http://dx.doi.org/10.3989/asclepio.2013.18

this treatise, the author says that they are meant for all users of Geometry, either practical or theoretical, among which are geometers, draftsmen, architects and engineers. The *General Trattato* had a great influence on Perez de Moya's geometry, especially on *Tratado de Geometria practica, y speculativa* (Alcalá de Henares, 1573) (Silva, 2011)[8]. This work disclosed in the Castillian language some of the main geometrical knowledge of the time. It is a landmark in the study of Geometry in the Iberian Peninsula during the last quarter of the sixteenth century (Silva, 2011, pp. 25-26, and pp. 76- 77), and it anticipated a new period of interest for that science in Spain which began with the opening in 1583 of the "Academia de Matemáticas" (Rey Pastor, 1934, p. 34).

3. OBJECTIVES, CONTENTS AND SOURCES OF JUAN PÉREZ DE MOYA'S GEOMETRICAL WORKS

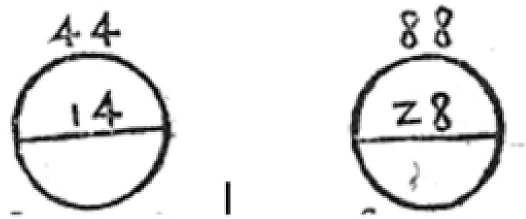
3.1. The firsts texts of geometry written by Moya: the *Fourth Book of Arithmetica practica, y speculativa* (1562)

The first notions of geometry given by Moya are included in the *Fourth Book* of his *Arithmetica practica, y speculativa*, and developed in a section consisting of three short chapters. The main objective of this text is to introduce the rules of land surveying, as the title of the last chapter shows (*Muestra la orden de medir tierras*). It was usual for the first Iberian printed treatises of arithmetics to include a brief section with some geometrical topics. It is the case, for example, of Juan de Ortega's *Compusicion de la arte dela arismetica y Juntamente de Geometria* (Ortega, 1512), of Gaspar Nicolas's *Tratado da Pratica Darismetyca* (Nicolas, 1519), and of Joan Ventallol's *Pràtica mercantívol* (Ventallol, 1521). Apart from the form, also the contents and the methods used are similar in these texts. The contents consist, in general terms, on the resolution of practical problems related with the calculation of the perimeter and the area of a plane figure. The problems posed in the case of the circle provide the approximate value 3 1/7 (figure 1) of π, which was also adopted by the authors mentioned above[9]. This value of π was already specified in the earlier texts of arithmetic and practical geometry (Smith, 1958, vol.2, p. 307) in particular in Pacioli's *Summa* (Pacioli, 1494, f. 30r)[10]. However, this was not the only option for the value of π. For example, the anonymous author of *Libro de Arismética que es dicho alguarismo* proposes to multiply the diameter of a circle by 3 1/3 to obtain its perimeter (Caunedo y Córdoba, 2000, p. 207).

Going back to Moya, we now present the statement and resolution of a problem concerning a round field the perimeter of which measures 44 staffs and the diameter is asked. He notes that the ratio between the circumference and diameter of any circle is 3 1/7 (*proporcion tripla sexquiseptima*):

> Es una tierra redonda, la qual tiene de circunferencia 44 varas, demando: ¿que tendra de diametro? Para saber esta, y sus semejantes, tendras por regla general que la proporcion de la circunferencia a su diametro es tripla sexquiseptima, y al contrario, del diametro a su circunferencia es subtripla sexquiseptima (Pérez de Moya, 1562, p. 310).

Figure 1. Relationship between the perimeter of the circumference and its diameter in (Pérez de Moya, 1562, pp. 310-311)



The procedures used by the cited authors are described in abacus's treatises (Simi, 2000, p. 156). They involve rules, such as the rule of three, the arithmetic concepts of square root and proportion, and simple geometric knowledges, such as the perimeter and the area of plane figures, and the theorems of Pythagoras and of Heron; for example Gaspar Nicolas proposes and solves forty-two geometric problems in such conditions (Nicolas, 1519, ff. 80r-94v).

Despite the similarity observed between Moya's *Fourth Book* and the other mentioned works, there is a significant difference in the introductory part of their texts. This part reveals the attention paid by our author to the definitions of the geometric objects with which he works, even when the text is directed for land surveyors, to whom the more theoretical issues possibly would not be important. This brief introduction to the Euclidean geometry gives an idea of the importance that Moya puts on the formulations and rigor. Euclid and Aristotle are authors mentioned in this *Fourth Book*. We stress that Moya was familiar with Ortega's *Compusicion de la arte dela arismetica y Juntamente de Geometria*, as is shown by the similarity between the two texts: (Pérez de Moya, 1562, p. 310) and (Ortega, 1512, ff. 196r-197r). But Moya mentions neither this title nor its author. As Sesiano already observed there is a work by Moya printed in 1563 with a reprint of the *Tratado subtilíssimo* by Juan Ortega (Sesiano, 1988, p. 38); we could verify that it is the *Sixth Book* in (Ortega, 1563, pp. 241v-254v).

On the other hand, some of the problems in *Fourth Book* do not appear in the works of the Iberian authors referred, but they are included in Pacioli's *Tractatus Geometrie*[11]. The similarities with Pacioli concern the type of elementary utensils used in the calculation of distances, heights and depths, and on the way of introducing geometry (Pacioli, 1494, f. 50v) and (Pérez de Moya, 1562, p. 316). Moya gives analogous definitions to those of Pacioli's *Summa*, and uses analogous terms to designate the same concepts; for example, *helmuayn* and *helmuariphe* in (Pacioli, 1494, ff. 1r-1v) and (Pérez de Moya, 1562, pp. 308-309). Nevertheless Pacioli's text is much more detailed and extensive[12].

3.2. Geometry with ruler and compass: the approach in *Fragmentos Mathematicos* (1568)

The first volume of *Fragmentos Mathematicos* is a text on geometry with different characteristics from the *Fourth Book* of *Arithmetica practica, y speculativa* (1562). It is much more extensive, entirely devoted to geometry, and related to problems that can be solved with the ruler and compass. According to Moya's letter to the readers (Pérez de Moya, 1568, "El Bachiller Ivan Perez de Moya a los lectores"), this work must constitute a fundamental set of tools to those who study other subjects.

The work is organized in forty-four chapters; the first forty providing useful tools to understand the remaining four. It starts such as *Fourth Book* with a set of definitions, postulates and ordinary sentences, or concepts, which are essential as a support of geometry:

> Y porque con mayor fundamento se pueda disputar y dar razon de esta arte [geometría], pondremos primero tres generos de principios, siguiendo la orden de Euclides, que son diffiniciones, peticiones, y communes sentencias (Pérez de Moya, 1568, pp. 2-3).

But while in Euclid's *Elements* the text is presented without further explanation, Moya clarifies the contents with geometrical or numerical examples[13]. And he uses Euclid's *Elements* to justify some practical constructions. In the *Fragmentos Mathematicos* we can find elementary subjects, such as the division of a line segment in equal parts by means of a ruled paper, but also some elaborate ones, such as justifications involving propositions of Euclid's *Elements*[14]. The constructions include the circumscription of a triangle by a circle, the quadrature of polygons and of the circle, the duplication of the cube, and a set of units of measures useful to cosmographers and geometers. Some of these units are already given in the works of Roman authors from the Classical Antiquity, which Moya cites: Columela, Vitruvius and Pliny. Moya mentions in particular the fifth book of Columela's *Res Rustica*, the third book (first chapter) of Vitruvius, and the second book (chapter 23) of Pliny. The titles of Vitruvius and Pliny's works are not mentioned, but it is highly probable that Moya was referring to *De architectura* and to *Historiae Naturalis*, because these are emblematic works of those authors.

The last chapters of *Fragmentos Mathematicos* are dedicated to issues of measurement: altimetry, planimetry and stereometry (for Moya these involves the measurement of corporeal things whatever the material of which they consist). Here the concept of measure is made explicit by Moya for the first time, referring it to the comparison with some known unit which he calls "famous measure":

> Medir una cosa no es otro, sino saber quantas medidas famosas contiene la cosa que se mide. Medida famosa dizen a una qualquier medida usada y notoria acerca de algunas o de muchas gentes (Pérez de Moya, 1568, p. 1).

His terms are very similar to those used by Tartaglia:

> Misurare alcuna quantita, non vuol inferir altro, che un voler trovar quante volte si ritrovi in quella alcuna famosa quantita, over qual parte, over quante parti sia di detta famosa quantita. Per famose quantita si debbe intendere per quelle specie di misure communamente usitate per le provincie, over citta, in tai misurationi (Tartaglia, 1560a, p. 1).

In the context of altimetry, the text shows how to determine heights, depths and inaccessible distances, using "regla status" and "two staffs". These instruments were not mentioned in *Fourth Book*. In planimetry, Moya teaches how to measure areas of regular polygons, the circle and the oval, and indicates rules to determine volumes (the "mass") and surface areas of solids (platonic and other non-regular solids). These studies have different applications, which Moya exemplifies in some practical problems. Sometimes, the rules proposed do not lead to exact values, and Moya knows it, but possibly their use would not require greater accuracy[15].

Fragmentos Mathematicos differ from *Fourth Book* by the diversity of authors cited in the former. Nevertheless, the references are not exhaustive and we can see some significant similarities with other texts which Moya does not denounce. We give three examples: the contents of the last common sentence are paraphrases from Aristotle's *Physics* (book III)[16]; the two formulas for determining the approximate area of an equilateral triangle when it side is known, $A=13xl^2/30$, and $A=433xl^2/1000$, are both from Cardano (Cardano, 1663, p. 117)[17]; the procedure for the division of a line segment into equal parts is already described in Ringelberg' *Ad mathematicen* (Ringelberg, 1531, pp. 481-482) and in Peletier's *De L'usage of Geometrie* (Peletier, 1573, p. 22). We have no doubt that Moya owes to Cardano a great part of the study on measurement of areas, mainly to polygons with more than

Asclepio, 65 (2), julio-diciembre 2013, p018. ISSN-L: 0210-4466. http://dx.doi.org/10.3989/asclepio.2013.18

four sides. Besides, Moya takes some applications of altimetry from Frisius, referring to a short treatise which is published together with the cosmography of Apiano (Pérez de Moya, 1568, p. 166)[18]. Moya does not indicate its title, but we can easily understand that it is *Libellus de Locorum describendorum ratione, et de eorum distantiis inveniendis, nunquam antehac visus* (Frisio, 1548, ff. 55r- 68v).

The questions about squaring the circle and duplicating the cube, which were under the attentive eyes of the mathematicians of the Renaissance, also took the interest of Moya. In the first case, he presented three approximate procedures (Pérez de Moya, 1568, pp. 69-71) − two of them adapted from Archimedes's *Measurement of the Circle* (propositions 1 and 2) (Dijkdteerhuis, 1987, p. 222), and the third one (Pérez de Moya, 1568, pp. 77-79) from Ringelberg[19]– but Moya does not mention the name of this author. The last process mentioned is based on a very simple construction given in the *General Trattato* (Tartaglia, 1560b, ff. 17v-22v) which Tartaglia associates to Bovelles (Carlo Bouile):

> Ma per non lasciar di narrar quanto che nella quadratura del detto cerchio ho trovato scritto, & massime di quele, che nella practica sono di qualche cõmodita, over utilita, Carlo Bouile in fin de l'opera sua, da un brevíssimo modo, over regola da ridure un quadrato in un cerchio (…) (Tartaglia, 1560b, p. 22r).

Tartaglia does not mention the title of Bovelles's work, but we can check that this subject is studied in the last folia of the *Géométrie en françoys* (Bovelles, 1511, ff. 39v-40r)[20]. The construction, also presented in Ringelberg' *Ad mathematicen* (Ringelberg, 1531, p. 485), consists in dividing the diameter of the circle in eight equal parts and taking the square whose diagonal is equal to ten of those parts (figure 2). The area of this square is approximately equal to the area of the circle.

The importance of doubling (obviously approximately) the cube is also clear in this work of Moya. For the construction, he recommends the lecture of the fourth book of Dürer's *Geometry*, stressing their efficacy:

> Alberto Durero enel quarto libro de su Geometria pone la orden practicalmente que se ha de tener para saber doblar, o tresdoblar el cubo, o cuerpo quadrado a forma de dado, porque es regla necessaria para muchas cosas mechanicas la quise poner aquí (Pérez de Moya, 1568, p. 85).

The mention of Dürer confirms the influence that this author had in Spain, mainly due to the practical features of his work (Vasconcelos, 1929, p. 74).

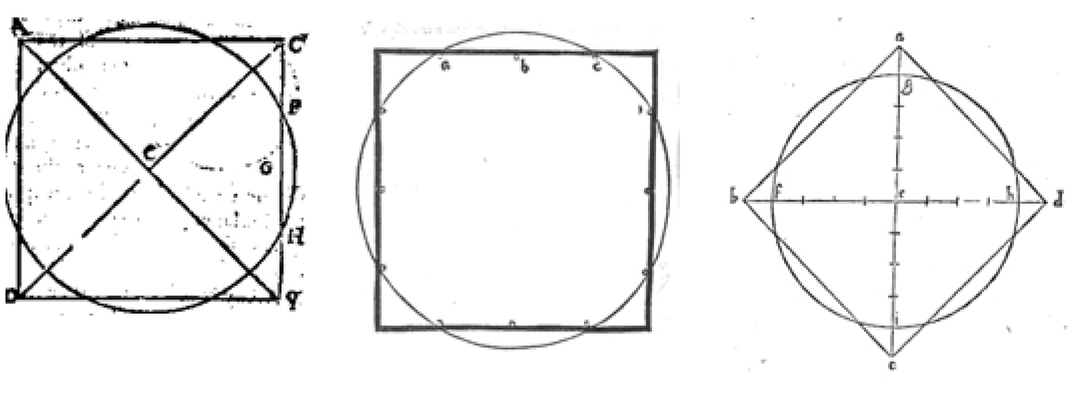
The influence of Tartaglia's *General Trattato* is clear in several passages of *Fragmentos Mathematicos*. For example, Moya proposes four procedures to divide a line segment in equal parts, two of which are given in the *General Trattato*; and in the case of the ovals, both the description and the construction of the figures are from Tartaglia (Tartaglia, 1560c, ff. 22v-23v) whose name Moya mentions (Pérez de Moya, 1568, p. 61)[21].

3.3. Moya's revisited geometry: The *Tratado de Geometria practica, y speculativa* (1573)

The *Geometria Practica y Speculativa* is the second volume of *Tratado de Mathematicas*. It is the most important geometrical text written by our author. Moya reorders the chapters of *Fragmentos Mathematicos* and, occasionally, reorganizes the subjects.

The volume is structured in four Books (*Libros*), divided in chapters, some of them consisting of articles. It begins with a summary indicating the contents in detail.

Figure 2. Quadrature of the circle in (Pérez de Moya, 1568, p. 78), (Ringelberg, 1531, p. 485) and (Tartaglia, 1560b, f. 22r)



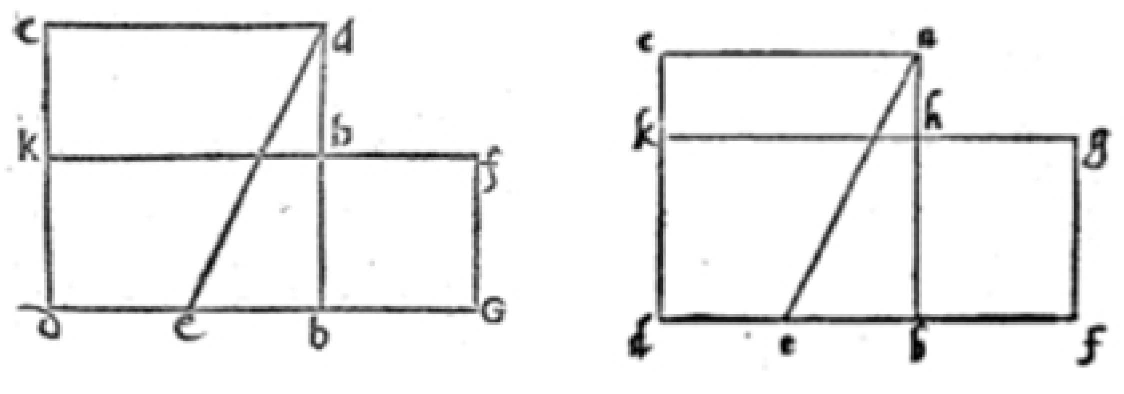
Moya uses a big part of the *Tratado* to explore practical applications of geometry (Books II-IV) reserving the first book for the theoretical foundation of the issues contained in the other three. The *Tratado de Geometria* includes all the contents of the *Fragmentos Mathematicos* that Moya believes to be essential, which makes this work more extensive and varied than the previous ones. Furthermore, it provides much more references to the sources, which are also more accurate. The division of the circle, the construction of polygons, and the excess between the inscribed and circumscribed polygons to the same circle are topics treated in *Tratado*, and not included in *Fragmentos Mathematicos*. But the opposite also happens: for example the construction of circles with area multiple or submultiple of a given circle presented in *Fragmentos Mathematicos* is not described in *Tratado de Geometria* (Pérez de Moya, 1568, pp. 72-77). It is true that *Tratado de Geometria* is more complex than the previous geometrical texts of our author, but its language is equally simple; however Moya recognizes that not all mathematical proofs are accessible to all its readers[22].

There are some important differences between the *Fragmentos Mathematicos* and the *Tratado de Geometria*. The most significant ones are probably the emphasis put in proofs of the *Tratado*, which are systematically founded on Euclid's *Elements*, and the distinction concerning practical and theoretical geometry. Moya identifies practical geometry with the use of the ruler and the compass, describing the latter's movements in detail and he associates theoretical geometry with Euclid's *Elements*. Furthermore, the *Tratado de Geometria* includes alternative strategies that minimize the construction work in some particular cases, and Moya stresses the importance of the generalization. For example, in *Fragmentos Mathematicos* the construction of the perpendicular to a line segment only concerns the midpoint of the line segment, but in *Tratado de Geometria* all possibilities are considered (Pérez de Moya, 1573, pp. 23-24). Moya explores the concept of perpendicularity in two distinct ways: one using a procedure justified by Euclid's *Elements* (Proposition I, 11) and the other offering a completely intuitive and practical construction, in which he simply uses a square. This last construction is analogous to the one which Tartaglia ascribes to the mathematically uneducated (*simplice natural*) (Tartaglia, 1560c, f. 3v).

Moya defines geometry similarly to Tartaglia, (Pérez de Moya, 1573, p. 5) (Tartaglia, 1560a, p. 1); and shares his opinion about the advantage of associating theoretical and practical geometrical knowledge. He begins his geometrical texts with definitions, postulates and common sentences, similarly to Euclid's *Elements*, following Campano's terminology. In *Tratado* he admits the existence of two types of points, corresponding to the interpretations of the *matematico* and the *natural*. He stresses that to the *matematico* the point is an abstract entity, indivisible and invisible, and to the *natural* the point is a very small sign that can be seen and divided into parts (Pérez de Moya, 1573, p. 6). This differentiation is adopted by Tartaglia, and other authors of the Renaissance such as Piero de la Francesca (Davis, 1977, pp. 41- 42) and Leonardo da Vinci (Arrighi, 2004, pp. 368-369), but we attest that Moya uses Tartaglia's terms to express these notions[23].

Moya follows Tartaglia when he describes the division of a line segment in mean and extreme ratio (or, as they say, *the division of a line segment according to the proportion that has the means and two extremes*). He explains the construction similarly to Tartaglia and uses the same figure (figure 3).

Figure 3. Division of a line segment according to the proportion which has the mean and two extremes: left hand-side in (Pérez de Moya, 1573, p. 28); right hand-side in (Tartaglia, 1560c, f. 7v)



Asclepio, 65 (2), julio-diciembre 2013, p018. ISSN-L: 0210-4466. http://dx.doi.org/10.3989/asclepio.2013.18

Moya also follows Tartaglia in many other occasions: for example in the definition of angle[24]; in the definition of the oval – a figure that attracted great interest among designers, painters, architects and craftsmen − commonly used in illuminations. In *Tratado de Mathematicas* he gives a construction of the oval (Pérez de Moya, 1573, p. 57), invented by Tartaglia in 1547 (Tartaglia, 1560a, ff. 11v-12r), which is more general than those of previous designers and architects. Moya refers to the difference between problem and theorem (Pérez de Moya, 1573, p.50), following Tartaglia (Tartaglia, 1560b, f. 1r), and stresses that all propositions dealing with triangles presented in the corresponding chapter of *Tratado* are problems.

The subjects related to circumscriptions between polygons and circles are introduced by means of seven definitions similarly to Tartaglia (Tartaglia, 1545, pp. 1-2) (Campano only gives two in Euclid's *Elements* Book IV); and Moya uses the ruler and the compass to solve the problems proposed in *General Trattato*, which the Italian mathematician solves in a way that he claims to be "more expeditious and more brief than that of Euclid". Nevertheless, our author gives a personal touch to this study by adopting an ordering of the subject different from the one of Tartaglia (see table 1).

There are two other topics in Moya's studies reflecting the great influence of Tartaglia's work. One is focused on the doubling of the cube and the other on the construction of an isosceles triangle with both angles of the base double of the angle at the vertex (Pérez de Moya, 1573, p. 53) (Tartaglia, 1560a, f. 14v). It is surprising that Moya does not allude to the relationship between such an isosceles triangle and a regular pentagon; he merely reproduces (Pérez de Moya, 1573, p. 56) an approximate construction of the regular pentagon that Bovelles exhibits in his *Geometrie practique* (Bovelles, 1551, f. 20r).

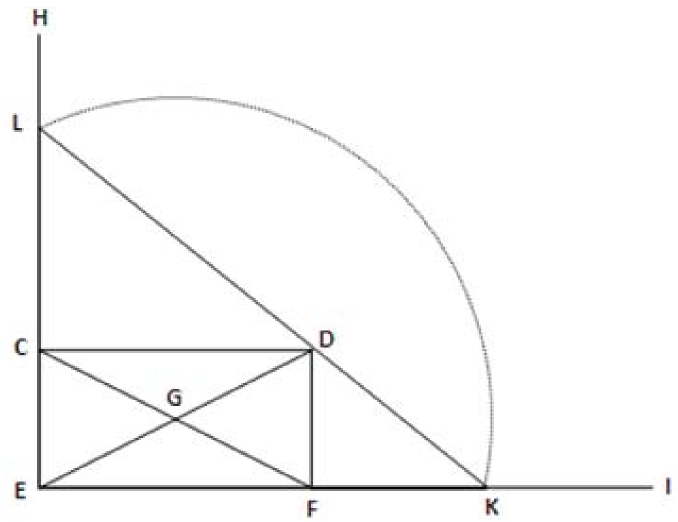
For the doubling of the cube, Moya explains the method of the "very ingenious" (*ingeniosissimo*) Tartaglia, believing that he, better than any other, shows a general rule to address this issue[25]. Moya starts with a cube of edge a, and then constructs the rectangle CDFE (figure 4) with sides a and $2a$, the diagonals of which intersect at G. He then traces the lines EI and EH, containing the sides EF and EC, and looks for the points K, belonging to EI, and L, belonging to EH, such that K, L and D are aligned, and K and L are both equally distant from G. The line segment FK thus obtained is equal to the edge of the required cube. The points K and L are discovered by trial; he draws a circumference centered in G and intersecting the lines EI and EH in K and L. Then, he gives suggestions to facilitate the final task, stressing that the points K and L are well determined only if the line segment KL contains D. If D falls above KL it is necessary to increase the opening of the compass, and if D falls below KL it is necessary to reduce that opening:

> (…) si tirando una linea de un punto a otro [o sea, de K a L], passar por el punto, o angulo D auras acertado, y si passare baxo, abre mas el compas y señala con el de la misma manera, y si passare alto, cierra el compas hasta tanto que se haga, que la linea que del un punto al otro se echare passe justamente por el punto D, como haze la linea KL (Pérez de Moya, 1573, p. 248).

Table I. Correspondence between the chapter with the study of propositions from Euclid's Elements Book four in (Busard, 2005, vol.1, pp. 142-159); in (Tartaglia, 1560c, pp. 13r-16r); and in (Pérez de Moya, 1573, pp. 53, 58-64)

Campano	Tartaglia	Moya
Elementos IV, 1	Cap. IX, 1	----
Elementos IV, 2	Cap. IX, 2	Cap. 32
Elementos IV, 3	Cap. IX, 3	Cap. 31
Elementos IV, 4	Cap. IX, 4	Cap. 27
Elementos IV, 5	Cap. IX, 5, 6	Cap. 23
Elementos IV, 6	Cap. IX, 7	Cap. 34
Elementos IV, 7	Cap. IX, 8	Cap. 33
Elementos IV, 8	Cap. IX, 9	Cap. 28
Elementos IV, 9	Cap. IX, 10	Cap. 24
Elementos IV, 10	Cap. IX, 11	Cap. 16. art. 5
Elementos IV, 11	Cap. IX, 12	Cap. 36
Elementos IV, 12	Cap. IX, 13	Cap. 35
Elementos IV, 13	Cap. IX, 14	Cap. 29
Elementos IV, 14	Cap. IX, 15	Cap. 25
Elementos IV, 15	Cap. IX, 16	Cap. 38
Elementos IV, 16	Cap. IX, 17	-----

Figure 4. Duplication of the cube: reproduction of the figure given in (Pérez de Moya, 1573, p. 249) (in Moya's figure the arc of the circumference is not seen. We introduce it for clarify)



To prove that this construction satisfies the required, Moya notes that the line segments FK and CL are the mean proportionals between *a* (= EC) and 2*a* (= CD), in other words:

$$\frac{a}{\overline{FK}} = \frac{\overline{FK}}{\overline{CL}} = \frac{\overline{CL}}{2a}.$$

And he uses Euclid's *Elements*, saying the following:

> Lo qual todo se demuestra por la duodecima diffinicion del quinto de Euclides, y por la treynta y seys del onzeno, que infieren en substancia, que siendo quatro lineas continuas proporcionales, la proporcion del cubo de la primera linea al cubo de la segunda, sera como la proporcion de la primera a la quarta de las dichas líneas (Pérez de Moya, 1573, p. 248).

These arguments are analogous to those of Tartaglia:

> Tutto questo si approva, & dimostra per la duodecima diffinitione del quinto, & per la trentesimasesta del undecimo, che in sostanza inferiscono, che essendo quattro linee continue proportionali, la proportione del cubo della prima, al cubo della seconda sara, come la proportione della prima alla quarta di dette linee (Tartaglia, 1560c, f. 21r).

As we know, the major works of practical geometry of the Renaissance included the study of measuring instruments, since this subject is closely linked with the resolution of problems of architecture, fortification, engineering, astronomy, cosmography and navigation[26]. We find measuring contents in Dürer, Peletier, Apiano, Frísio and Finé, for example, and it is not surprising that Moya knew their works, because they were very well represented in the Spanish libraries. In *Aspectos de la Ciencia aplicada en la España del Siglo de Oro*, Maroto and Piñero say that at the beginning of the sixteenth century the study of instruments constituted a significant part of the work of some Spanish authors[27]. But we stress that the only Spanish author cited by Moya on this subject, is "doctor" (*doctor*) Aguilera (Pérez de Moya, 1573, p.33), referring to his work *Canons astrolabii universalis* published in 1554[28].

As Maroto and Piñero note, there are important similarities between Moya and Finé concerning the study of altimetry instruments and its applications (Maroto and Piñero, 2006, pp. 341-349). But we can add that there are also significant differences. Actually, for example, Moya graduates the geometric quadrant from 1 to 12 (Pérez de Moya, 1573, p. 94), while Finé graduates it from 1 to 60 (Finé, 1587, f. 29r) which allows more accurate measurements. Furthermore Moya generally uses in the applications different values from Finé and he refers to different versions of Euclid's *Elements*. Moya is consulting Campano's version, and Finé probably follows Zambert's version[29].

The division of the circle into equal parts only using the ruler and the compass was studied by the authors of the Renaissance. Sometimes, they included in their texts approximate constructions, even when exact constructions were possible (Simi, 2002, p. 14). Moya exposes the subject for the first time in his *Tratado de Geometria* explaining how to divide the circumfe-

Asclepio, 65 (2), julio-diciembre 2013, p018. ISSN-L: 0210-4466. http://dx.doi.org/10.3989/asclepio.2013.18

rence in 2, 6, 8, 12, 16, 24, 32 and 36 equal parts. The study begins with the construction of the perpendicular bisector of a cord and uses the procedure and the figure presented in the *General Trattato* (Tartaglia, 1560c, ff. 10r-10v); the only difference is in the version of Euclid's *Elements* used, because Moya follows Campano while Tartaglia follows his own *Euclid's Elements* version (Tartaglia, 1545, f. 65v)[30].

Moya knew that, with an opening of the compass equal to the radius of the circumference, he could determine the sixth part of the circumference and that this would allow him to divide it in twelve and in twenty-four equal parts. A similar reasoning would have enabled him to divide the circumference in 8, 16 and 32 equal parts, justifying it by Euclid's *Elements* book IV. However, he used a different technique beginning with the division of the quadrant in three parts, which leads to an approximate division of the circumference.

Our author does not explain why he uses approximate procedures when there are exact constructions for the same purposes; but we stress that in all of them the compass has a fixed opening which is equal to the radius of the circumference that he wants to divide, and that they provide very good approximations (the error is nearly one thousandth part of the radius of the circle considered). Furthermore, Moya also indicates a construction (obviously approximate) for the division of the circle in 36 equal parts, using the fourth division of the circumference into three equal parts (Pérez de Moya, 1573, p. 43). In the second volume of *Fragmentos Mathematicos* (1567) a procedure was already referred, which gives an approximate division of a circle in 36 equal parts, using its division in nine parts (Pérez de Moya, 1567, p. 436). Moya ascribed it to Rodrigo Zamorano and considered it a good invention. Nevertheless, in *Tratado de Geometria* (1573) he indicates another construction, reproduced from Porres de Osorio (probably from his *Nuevas Proposiciones Geometricas*), which leads to a better approximation than the former[31].

The use of the compass with a fixed opening in the constructions was a real challenge in the mid-sixteenth century. Tartaglia devotes to this subject a chapter of the Fifth Part of *General Trattato* (Tartaglia, 1560c, ff. 22r- 22v), using this technique in the solving of several problems of Euclid's *Elements*[32]; and the emphasis given by Moya to this kind of constructions reflects, once again, the influence he has had from Tartaglia.

3.4. The last mathematical work of Moya: The *Principios de Geometria* (1584)

Moya's *Principios de Geometria* consists of two "Books" (*Libros*). The first book begins with definitions, petitions and ordinary sentences as in his previous geometric works. And it includes fundamental knowledge as, for example, the procedure for dividing a line segment in equal parts and for constructing different kinds of triangles, a square, a regular pentagon, a circle, an oval. It also teaches the division of the circle in equal parts and the squaring of the circle. The title of the second book is "It concerns things belonging to the kind of measure called Planimetry which belong to measuring and dividing land" (*Trata de cosas pertenecientes al genero de medida, que dicen Planimetria que pertenece al medir e dividir tierras*). The complete title of the book points to its recipients: men who study liberal arts and surveyors[33]. Moya advises the reader that its content is neither original nor comprehensive and that it is a part of his *Tratado de Geometria*.

Principios de Geometria has a reduced size, allowing easy transport and handling, and its content is more practical than the previous *Tratado de Geometria*. The authors' main concern is to indicate the simplest process for solving each proposed question, as shown in the following remark:

> Para medir triángulos, ay tantos modos, y primores que dezir que quererlos referir aqui seria confundir los entendimientos de algunos medidores, con los muchos preceptos, los quales por averlos puesto en otro volumen, solo pondre una regla general para medir cualquier triangulo de qualquiera suerte y genero que sea, con solo la noticia de sus lados (Pérez de Moya, 1584, vol.2, chapter X).

In this work, Moya keeps a similar style to *Fragmentos Mathematicos* and *Tratado de Geometria*, but he is now more interested in the applications of geometry to the solving of practical problems and in the details related to the construction of rudimentary tools of measurement. Here, the problem follows immediately the corresponding topics, showing its practical use; Moya does not dedicate to them a special section in the end of the work, as he had done in his other geometrical texts.

One topic discussed is the division of the circle. Moya gives the exact construction for dividing the circle into three equal parts and constructions for the approximate division of a circle in 5, 7 and 11 parts, which efficiently work; he takes from Dürer the division in 7 equal parts (Pedoe, 1976, p. 68).

Moya refers to some of his sources. Euclid is the most cited author and while he does not always clarify the name and the version of the *Elements* that he uses, he usually mentions the book and the proposition used. The first book of the *Elements* is the most often cited but there are also allusions to books III, IV and VI (in the first book of *Principios de Geometria*) and to books II and V (in the second book). The only new references are Serlio (1475-1554) who is the author of *Tratado de Architettura*[34], and Vignola (referred by Pinola in the text) who is author of *Le due regole della prospettiva*. Moya citesVignola in the context of architecture. On the other hand, many of the references mentioned in his other works are not included here, certainly because Moya did not think that they were useful to the recipients of this work.

4. CONCLUSIONS

Since the writing of his first geometrical text until the printing of *Principles of Geometry* (1584), Moya developed and consolidated his sources, selecting from them the contents that better match his purposes: which are the disseminating of geometrical principles to those who used geometry as a tool.

The geometrical works of the sixteenth century that mainly influenced him were from French and Italian authors, such as Peletier, Finé, Bovelles, Cardano and Tartaglia, but to this list should be added (and it is not exhaustive!) the names of Dürer, Ringelberg, Frisius, and the Spanish Aguilera. Sometimes, Moya indicates incompletely the titles of the texts consulted. For example, he simply says Forcius's *Geometry*, Dürer's *Geometry*, Frisius's *Tratado of the description of the places*, Aguilera's *Astrolabe*.

We are sure that among the geometrical works of the sixteenth century his main source is Tartaglia's *General Trattato* (1560), and especially the third and the fifth parts. However, Moya mentions profusely other authors such as Aristotle, Archimedes, Ptolemy, or Columela, Pliny and Vitruvius, who attracted the interest of scholars of the Renaissance. Besides Aristotle's *Physics*, *De Cælo & Mundo* and *Meteo*, and Archimedes's *Measurement of the Circle*[35], Moya particularly specifies the fifth book of Columela's *Res Rustica*, the third book of Vitruvius (first chapter), and the second book of Pliny (chapter twenty three) (Pérez de Moya, 1568, p. 237). The titles of Vitruvius and Pliny works are not given, but probably they are *De architectura* and *Historiae Naturalis*, because these were emblematic works of those authors. Euclid's *Elements* is the text more often cited by Moya, especially in the version of Campano. Nevertheless, we recognize that Moya knew Euclid's *Elements* in the version of Zamberto, because he refers to it (Pérez de Moya, 1562, pp. 43 and 145). He may have read the 1505 edition, or one of the editions containing both Campano's and Zamberto's versions, since they were then very well-known in Spain, and several copies of which are still extant in Spanish libraries. For example, the Library of Salamanca has the editions of 1505, 1517 and 1546; the Library of Catalonia has the editions of 1537 and 1546, and the Library of Seville has the edition of 1517.

The interest that Euclid's *Elements* raises in Moya and the importance that he attaches to them were already evident in *Fragmentos Mathematicos* (1568), where he reveals his intention to write some notes about this work, saying "as we shall show in our annotations on Euclid God being served" (*como mostraremos en nuestras annotaciones sobre Euclides siendo Dios servido*). The definitions, postulates and common sentences with which Moya begins his geometrical texts are from Euclid's *Elements*, as well as many of the propositions that he proves or simply states.

As it is known, the first printed edition of Euclid's *Elements* in castilian is due to Rodrigo Zamorano and dates from 1576. Therefore, Moya when referring often to the propositions of Euclid's *Elements* may be considered the major spreader of this important work in Spain.

5. ACKNOWLEDGMENTS

I am very grateful to Antoni Malet and to Carlos Sá for their thoughtful and detailed suggestions.

NOTES

1 Research funded by the European Regional Development Fund through the programme COMPETE and by the Portuguese Government through the FCT – Fundação para a Ciência e a Tecnologia under the project PEst-C/MAT/UI0144/2011.

2 See the letter addressed by Moya to readers (Pérez de Moya, 1573, "El Bachiller Ivan Perez de Moya a los lectores").

3 When Moya published *Compendio dela regla dela cosa*, existed just two texts printed in the Iberian Peninsula, which included subjects related to the study of algebra. One was Marco Aurel's *Libro primero de Arithmetica Algebratica* (Aurel, 1552), and the other was Bento Fernandes's *Tratado da arte de arismetica*, (Fernandes, 1555) – But these two works study many other subjects in addition to algebra (Silva, 2008). Moreover, regarding the Iberian manuscripts containing algebra, prior to 1552, we only know Ms 71 from Sant Cugat, which is a Catalan commercial text of arithmetic and algebra, probably written by Joan Ventallol around 1520 (Docampo, 2008, p. 90) and (Docampo, 2009, pp. 123-124).

4 The printing of the first book was delayed by the large number of its figures, and therefore the second was printed before it.

5 See the letter wrote in December of 1567, in (Pérez de Moya, "El Bachiller Ivan Perez de Moya a los lectores").

6 This is a rare book of which only a copy is known. It is kept in the National Library of Portugal (Res 6553P). A brief study of this work is given in (Silva and Malet, 2008).

7 See (Silva, 2011, p. 210) footnote 456.

8 See (Silva, 2011, pp. 305-307, 310).

9 See (Ortega, 1512, f. 197r), (Nicolas, 1519, f. 84) and (Ventallol, 1521, f. 123v).

10 According to (Smith, 1958, vol.2, p. 307), this value became satisfactory after being known the second proposition of Archimedes's *Measurement of the Circle* (c. 225 BC) in which the ratio between the area of a circle and the square of it diameter is approximately equal to 11:14.

11 See, for example, (Pacioli, 1494, Distinctio septima, f. 50v; Distinctio prima, f. 10r; Distinctio septima, f.52r; Distinctio tertia. f. 15v, and Distinctio quarta, f. 30r and followings).

12 Pacioli dedicates 76 folios of the second part of his *Summa* to geometrical subjects including in it definitions and propositions from the first, second and sixth books of Euclid's *Elements*.

13 The example given for the first conception involves three segments and the example given for the sixth conception is numerical (Pérez de Moya, 1568, pp. 15-16).

14 Moya referes to *Elements* I, II, III, IV, VI, X, XII, XIII e XIV.

15 For example to the area of the regular pentagon he gives $A=r\times P/2$, r being the radius of the circumscribed circle, and to the perimeter P of the pentagon he gives $A=3l^2-l/2$, or $A=5056l^2/2936$, l being the side of the pentagon.

16 It says: "Continued greatness cannot be infinitely increased but may be infinitely decreased" (Pérez de Moya, 1568, p.17). See Aristotle in (Heath, 1970, p. 110).

17 Cardano says the following: «Pro mensurando trigono æquiltero quadrabis latus eius, & productum multiplicabis per 13, & divide per 30. (...) si autem velles præcisius multiplica per 433. & divide per 1000.» (Cardano, 1663, p. 117). Cardano's work was first published in Milano, in 1539, Milano, by Antonins Castellioneu. The first procedure is also given in (Finé, 1587, ff. 50v - 51r).

18 Moya says: "I took all the major part of this article from Cardano" (*Toda la mayor parte deste artículo saque de Cardano*) (Pérez de Moya, 1568, p. 166). Besides, Moya mentions Frisio when he explains how to use two staffs in altimetry.

19 For biographical notes about Ringelberg see (Pereira, 2004, pp. 201-202).

20 The *Géométrie en françoys* (1511) was probably the first geometrical work printed in French.

21 Moya mentions the italian geometer but he does not indicate the name of the work that he consulted (Pérez de Moya, 1568, p. 61).

22 See (Pérez de Moya, 1573, "El Bachiller Ivan Perez de Moya, al letor").

23 Also in (Rojas, 1598, ff. 44r-44v).

24 See (Pérez de Moya, 1573, pp. 9-10) and (Tartaglia, 1560 a, ff. 1r-1v).

25 See how Tartaglia deals with this subject in (Tartaglia, 1560b, ff. 20v-21r); compare with (Pérez de Moya, 1573, p. 249).

26 For the different instruments used in architecture in the sixteenth century, see (Gessner, 2010, pp. 11-12).

27 On the texts that include problems of altimetry we mention: Juan Martín Población's *De usu astrolabi compendium* (Paris, 1520), Juan de Rojas's *Commentariorum in Astrolabium quod planisphaerium vocant* (Paris, 1551), and in the late sixteenth and early seventeenth Diego de Álava y Viamont's *El Perfecto Capitán, instruído en la disciplina Militar, y nueva ciencia de la Artilleria* (1590), and García Céspedes's *Libro de instrumentos nuevos de Geometria muy necessarios para medir distancias, y alturas, sin que intervegan numeros, como se demuestra en la practica* (606). For further detail see (Maroto and Piñero, 2006, p. 254). See also (Soler Selva, 2002, pp. 119 e 129).

28 This is not the first time that Moya cites Aguilera, as shows (Pérez de Moya, 1567, p. 172).

29 Moya refers to proposition 46 from Euclid's *Elements* book I; Finé indicates it as the 47 of the same book. In Campano's version of Euclid's *Elements* the proposition number 47 is the converse of the Pythagorean theorem.

30 Moya claims to Campano's *Elementos* III, 29; for this proposition see (Busard, 2005, vol.1, p.131).

31 In (Pérez de Moya, 1573, "El Bachiller Juan Pérez de Moya, al letor") says: "Los modos de dividir una circunferencia de un circulo en mas de doze partes, que pusimos enel capitulo catorze, del primero libro de Geometria, son de Juan de Porres Ossorio de Mendoça, Legista, natural de Mexico, vecino de Ciudad Real". We could not determine the date on which this work was printed. (Altamirano, 2008, vol. 1, p. 70) says that it was known about 1580, but as we have shown Moya knew it before that date.

32 In 1553, Giovanni Battista Benedetti (Venezia, 1530 – Turim, 1590) publishes in Venice a work entirely devoted to this topic, entitled *De resolutio omnium Euclides problematum aliorumque (...) una tantummodo circuli data apertura*.

33 The title is "Principles of Geometry from which the students of Liberal Arts can take advantage, and also for every man whose profession requires the use of the ruler and the compass. Containing the procedures for measuring and dividing lands" (*Principios de Geometria de que se podran aprovechar los estudiosos de Artes Liberales, y todo hombre que su officio le necessitare a tomar la regla y compas en la mano. Con el orden de medir, y dividir tierras*).

34 This work has by five books which were published together for the first time in 1584, in Venice, under the title *Tutte l'Opere d'architettura et prospettiva*. However, Serlio's work was already well-known, and it was even translated into other languages. See, for example, (Serlio, 1545).

35 See (Pérez de Moya, 1568, pp. 5, 9, 243), and (Pérez de Moya, 1568, pp. 7, 166, 220, 224, 242, 262).

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